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# Derivatives

**Peter Moles**

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# Derivatives

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# Introduction

This elective course covers one of the core areas of market finance, namely derivatives. The major classes of derivatives – forwards, futures, options, and swaps – are key instruments for allowing market participants to transfer and mitigate risks and to speculate on future asset values. The growth in the size and diversity of derivatives markets testifies to their importance within the financial system. Furthermore, the theory of option pricing is one of the key ideas in finance for which Myron Scholes and Robert Merton were awarded their Nobel Prize in 1997.\* The Black–Scholes option pricing model has been described as ‘*the workhorse of the financial services industry*’. Understanding derivative pricing is an important element for financial engineers when seeking to address problems in finance.

Financial futures are one of the most heavily traded markets in the world, with futures exchanges existing in all major countries. Since the mid-1970s over seventy futures (and options) exchanges have been established. They are organised markets for exchanging a wide variety of financial and business risks, ranging from interest rates across to insurance and, latterly, weather. The volume of transactions and the types of instruments available to speculate on and manage risk continues to increase as new uses are found for futures.

The development of a theory to price contingent securities has had major ramifications for the financial services industry. Option markets, both formal markets (as with futures) and over-the-counter trading between principals, have expanded dramatically following the introduction of a working model for their pricing. Option pricing is complex with a number of factors determining their value. Many financial transactions include option-like elements. In addition, some problems in corporate finance can also be best understood in terms of option theory. This course provides a conceptual understanding of how options are priced and how they can be used for a wide range of risk management and other uses by financial practitioners.

Swaps are one of the newest developments in the derivatives product set and have become an important component of derivatives markets. The pricing of swaps illustrates how financial securities are valued in a competitive market. Swaps are a key tool for asset-liability managers for all types of firms and complement the derivative instruments available in futures and options markets.

A large part of the role of finance, the actions of the financial specialist and the operations of the financial department within firms, are devoted to handling, controlling, and profiting from risk. Hence this course emphasises how market participants manage and exploit financial risks using derivatives. Of course, such instruments can also be used for speculation or arbitrage.† But it is the ability of

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\* Nobel Prize in Economic Science, 1997, for ‘a new method to determine the value of derivatives’

† We only need to look at the activities of Nick Leeson of Barings Bank fame to see how an arbitrage strategy can be easily turned into a speculative one! He fraudulently undertook highly speculative transactions when he was supposed to be involved in low-risk arbitrage activity. As a result, Barings Bank collapsed in 1995.

derivatives to modify risks that has helped place these instruments at the centre of current activity in the global financial markets.

Before starting this course, the student is expected to have some prior knowledge of the fundamentals of finance and, in particular, time value of money methods and an understanding of statistical concepts. The level of knowledge required is that which it is necessary to have in order to successfully complete a course in finance. It is also strongly recommended that students have taken *Financial Risk Management* which covers the sources of financial risk and methods of risk assessment.

## Arrangement of the Course

The Modules that go to make up *Derivatives* fall into the following topic areas:

	<b>Topic area</b>	<b>Modules</b>
One	<b>Introduction to the Derivatives Product Set:</b> Introduces the fundamentals of derivatives and their pricing, the different types, and their uses	1 and 2
Two	<b>Terminal Instruments</b> The different types of terminal derivatives: forwards, futures and swaps	3 to 5
Three	<b>Options</b> The nature, types, pricing and uses of options	6 to 10
Four	<b>Using Derivatives and Hedging:</b> Risk management using derivatives	11, 12

The initial modules (Part One) introduce the different types of derivatives, namely forwards, futures, swaps and options, how they are used, and explains the way in which they can be valued.

The discussion then proceeds to cover in detail the mechanics and use of the different terminal instruments, that is, forwards, futures and swaps (Part Two), and options (Part Three) principally as risk management tools – since this is a prime justification for the growth in derivatives markets – and shows how they can be incorporated into the process as a means to transfer and control risk. The application of these tools then follows (in Part Four), together with some of the inevitable complexities that result from this process.

In presenting the text in this way, the aim is it provides a comprehensive and logical approach to what is a complex subject.

## Approach and Key Concepts

Derivative pricing is a complex subject. The text presents the different derivatives product set elements in rising order of complexity. Whilst this is useful in developing a good understanding of how the derivatives product set works, it does have some disadvantages in that material on one subject (for instance, the cost of carry

model used to price forwards and futures) is presented in different parts of the elective. As a result, we would encourage students to look at alternative ways to approach the text.

A basic premise of the material is that it is orientated towards the needs of a market user, with a strong emphasis on using derivatives for risk management purposes. Of course, as is explained at different points, these instruments can be used for other objectives – for instance, speculating and spreading.

As a course, it concentrates on the methodological and operational issues involved in using derivatives. That is, it is technique based and emphasises the mathematical, financial, or engineering approach to these instruments. Market users can – and do – use these instruments without such knowledge. But seasoned practitioners will agree that gaining the understanding of derivatives that this course provides will assist you in using these instruments wisely.

As a subject derivatives introduces ideas that are central to modern financial theory and practice. Daily, and all over the world, practitioners are putting to use the models described in this course to manage the ongoing financial risks in the organisations for which they work. For instance, the ideas behind option theory and arbitrage pricing are central to managing the risks of contingent cash flows. It is a prerequisite for anyone wishing to pursue a career in financial services or become a financial specialist to gain an understanding of derivatives markets and pricing.

## Assessment

As is customary with this programme, you will find self-test questions and cases at the end of each Module. The answers are given at the end of the text. Also, there are two pro-forma exams of the type it is necessary to pass in order to gain credit from this course. The exam assessment is based on the following criteria:

Section	Number of questions	Marks obtainable per question	Total marks for the section
Multiple choice questions	30	2	60
Cases	3	40	120
			180

# Acknowledgements

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# **PART I**

## **Introduction to the Derivatives Product Set**

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**Module 1 Introduction**

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**Module 2 The Derivatives Building Blocks**

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## Introduction

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### Learning Objectives

Derivatives have become an important component of financial markets. The derivative product set consists of forward contracts, futures contracts, swaps and options. A key issue is how prices for such derivatives are determined. The ability of market participants to set up replicating portfolios ensures that derivative prices conform to no-arbitrage conditions. That is, the prices cannot be exploited without taking a risk. Replication also explains how derivative claims can be manufactured to order.

The principal justification for the existence of derivatives is that they provide an efficient means for market participants to manage risks. But derivatives also have other uses such as speculation and the implementation of investment strategies.

After completing this module, you should:

- know the history of the development of derivatives, namely that:
  - there is early historical evidence for forward and option contracts
  - futures contracts were developed in the 19th Century and that financial futures were introduced in 1973
  - swaps were first traded as recently as 1981
  - new derivative products continue to be developed to meet specific needs of market participants
- know that derivatives are designed to manage risk, usually the price or market risk of the underlier that arises from uncertainty about the underlier's value in the future. In particular, that:
  - market participants who need to buy in the future are exposed to the risk that prices may rise before they can buy. This exposure to price risk is known as buyer's risk

- market participants who need to sell in the future are exposed to the risk that prices may fall before they can sell. This exposure to price risk is known as seller's risk
- be able to differentiate between the different elements of the risk management product set, namely forward contracts, futures, swaps, and options;
- understand how prices in financial markets are maintained in proper relationship to each other through arbitrage;
- be aware that arbitrage relationships rely on the Law of One Price and how imperfections in the way real markets operate can limit the applicability of the law;
- understand that the payoff of derivative instruments can be replicated using combinations of fundamental financial instruments;
- understand how in an efficient market the prices of derivatives, which can be replicated using fundamental financial instruments, are determined through arbitrage-free relationships;
- know the main uses for derivatives, namely:
  - risk modification
  - hedging
  - speculation
  - spreading
  - arbitrage
  - lowering borrowing costs
  - tax and regulatory arbitrage
  - completing the market
- be aware that the main justification for derivatives is that they enable market participants to efficiently transfer risks.

### 1.1 Introduction

In 1995, Nick Leeson a trader at *Barings Bank* made the headlines when it became public knowledge that, unknown to his bosses, he had run up losses of US\$1.3bn through dealing in derivatives. Prior to this, many people had been unaware of the importance of derivatives in the financial system and their capacity to generate profits or (in Leeson's case) disastrous losses. Derivative is the generic name for a set of financial contracts that include, forward contracts, futures, swaps and options. The term derivative comes from the fact that the instruments obtain their value (derive it) from the behaviour of more basic underlying variables. Hence derivatives are also often referred to as contingent claims. The underlying variables can be a specific asset or security, index, commodity, or even the relationship between different assets. The main classes of instruments are forward contracts, futures, swaps and options. Later modules of this course will examine each of these instruments in detail.

The number, type and variety of derivative contracts has expanded greatly since the introduction of the first exchange-traded instruments in the early 1970s. Since then, instruments have been introduced to manage the risks in interest rates,

currencies, commodities, equities and equity indices, credit and default risks, and other financial risks. This increased variety, coupled to a wider use of derivatives by market practitioners to address a variety of problems, has meant an explosion in the volume of outstanding contracts.

While the current interest in the use – and abuse – of derivatives has been a recent phenomenon, the commercial world has employed derivative contracts since the dawn of trade. The increased use of financial derivatives, that is instruments used to manage or speculate on financial risks, can be traced back to the introduction of financial futures in 1972 by the *Chicago Mercantile Exchange* (CME) and by the *Chicago Board of Trade* (CBOT) and options on company shares by the *Chicago Board Options Exchange* (CBOE) the following year. The CBOE is a subsidiary of the CBOT, an exchange established in the 19th century to trade derivatives on agricultural products. By introducing financial futures, the CME was responding to a demand by financial markets for better ways to manage risks. By offering exchange-traded options, the CBOE made available contracts that provided insurance against future uncertainty.

Since 1973 the use of financial derivatives has snowballed and many new financial derivatives exchanges have been established. Not only has the volume of transactions increased but the type and complexity of the instruments themselves has increased dramatically. For instance, the original types of options traded at the CBOE are now referred to as standard options to distinguish them from the exotic options that have since been introduced.

Derivatives were introduced into commerce as a necessary tool for merchants to handle risks. The principal risk that they are designed to manage is the price risk or market risk of the underlier (the asset, security or variable that is the basis of the derivative contract). The earliest form of derivative is the forward contract, which is simply a purchase/sale agreement where the implementation or settlement of the contract is deferred to some mutually agreed date in the future. In a normal contract the purchase/sale leads to an immediate transfer of the contracted element from the seller to the buyer, that is ‘on the spot’ and hence are called spot contracts or cash market contracts. With the forward contract, the transfer of the underlier is deferred to a mutually agreed date although the price (and other features such as quality and quantity) is agreed today. The attractions for both buyer and seller are obvious: by trading now the buyer is guaranteed the price at which he can purchase. In the same way, the seller is guaranteed the price at which he can sell in the future. This arrangement makes a lot of commercial sense and evidence from earliest history suggests that fixing a price for future delivery was an important element in commercial activity. Early evidence of the prevalence of such contracts comes from the ancient Assyrian commercial code, which included laws governing the writing and enforcement of such contracts. There is also evidence from as early as 2000BC of forward dealing in India. Historians have uncovered evidence that ancient Rome had a market in such forward contracts for wheat, the staple commodity food for the city.

In the 15th century, historians have documented that Antwerp was the centre of a sophisticated forward currency market linked to the Flanders cloth trade. Mer-

chants due to receive or make payments in one of the many different currencies that circulated in Europe at the time were able to fix in advance the exchange rate for conversion (for example from florins to marks) so as to eliminate the risk. In fact Thomas Gresham, the English businessman, established a bourse (or exchange) in London in direct imitation of those that existed in Antwerp. This later became the *Royal Exchange*. Gresham's initiative was an early example of the commercial competition for the management of risks. In the 18th century, terminal markets at dockyards and other transit points were the focal points for dealing in forward contracts. Merchants with goods being shipped to the port would be concerned that their cargoes would temporarily upset the demand and supply balance. To counter this, they would 'sell forward' part or all of their products for delivery when the ship docked.

In the mid-19th century, Chicago, Illinois, had become a centre for the mid-West. Its proximity to the Great Lakes and the grain growing plains meant that farmers shipped their produce to the city. The seasonal nature of production meant that prices for grain rocketed in the spring but collapsed after the harvest. In 1848, merchants in the city gathered together to find a better way of organising the grain trade. As a result, the *Chicago Board of Trade* was created. Over the next few years, the technology of forward contracts was refined. The result was the development of futures contracts. While economically the same, these differed from forward contracts by the fact that they not only managed the price risk in the underlier but that they eliminated the credit risk that exists in forward contracts. The benefit of a forward contract depends entirely on the willingness of both parties to honour the agreement. If the market price changes substantially, there is a strong incentive for the buyer (seller) to renege on the agreement and buy (sell) in the spot market. The development of futures solved the performance risk problem by requiring each party to collateralise their position. Futures have allowed a tremendous expansion of the market in forward transactions since there is no longer a requirement to check the soundness of the party with whom one is dealing.

Unlike forwards or futures contracts, options allow the buyer, known as the option holder, the right to terminate the agreement and hence are more flexible. Like forward contracts the first use of option contracts pre-dates written records. There is an account by Aristotle of Thales, a philosopher in ancient Athens, about the use of options. While the account is meant to show the benefits of an understanding of philosophical ideas, the story itself shows that the use of options for commercial purposes was well established. The story is that, stung by critics as to why he was poor, Thales used the insights he had developed through philosophy to make himself a considerable fortune. Observing that the forthcoming olive harvest was likely to be a good one, he travelled around Attica making contracts with olive press owners to hire their facilities in the autumn. As he had little money, the contracts involved his being given the right of first use for the press at a given price. He paid a small amount of money for this option. In the event as he had anticipated the harvest was abundant and Thales was able to exercise his option and hire out the presses at a profit to growers, making him rich in the process. While Aristotle's account may be exceptional, there is good historical evidence elsewhere. For instance during the Shogun era, the Japanese silk trade made frequent use of option contracts. Options also are often written into

commercial contracts. For instance, many contracts allow the buyer to cancel delivery in exchange for a fee.

Prior to the initiative taken by the *Chicago Board of Trade* in 1973, options on financial instruments had been traded in financial markets, but were considered esoteric and of little significance. The existence of traded options plus the happy coincidence of the publication of the Black–Scholes option-pricing model (BSOPM) greatly accelerated the expansion of the market in financial options. BSOPM provided a mathematical solution to the pricing of options based on two important premises. First, that the value of an option can be modelled by looking at a replicating portfolio which has the same payoffs as the option, and second, the importance of arbitrage forces in an efficient market. In the 1970s, financial institutions introduced options on an ever-wider range of financial assets and sectors: currency options, options on stock and other indices, options on interest rates and debt securities to name but a few. In the 1980s financial engineers, that is mathematically adept modellers of such contingent claims, were able to develop a range of options with non-standard terms and conditions. These ‘exotic’ options offered features such as average prices (known as average rate options), or fixed payoffs (binary options), or under certain conditions ceased to have a value (that is, they were ‘knocked out’), and many more. More recently, a second generation of exotic options has been created with names such as perfect trader that greatly expand the opportunities available. Today, financial market users can find options to manage all sorts of different risk characteristics. And if they cannot, they can ask a financial institution to create one that exactly meets their needs.

### Swiss Re and Mitsui Sumitomo Insurance Swap Catastrophic Risks

In August 2003, *Swiss Re*, the reinsurance company, and *Mitsui Sumitomo Insurance* of Japan entered into one of the world’s first catastrophe risk swaps. The US\$100 million transaction between the two insurance companies allows each company to reduce its exposure to natural disasters, known in the insurance industry as catastrophic risk, in its core market by passing on this risk to the other party.

Under the agreement announced by the two firms, *Swiss Re* swapped US\$50 million of potential insurance losses from North Atlantic hurricanes with the same amount of protection given by *Mitsui Sumitomo Insurance* for a Japanese typhoon.

According to a spokesperson at *Swiss Re* the key attraction was to swap future potential insurance payouts on rare but devastating events. The likelihood that the event would occur is about 2 per cent; that is, there is an expectation that there will be one such event every 50 to 100 years. However, if such an event happened, both insurers would be exposed to very large losses. Such events are known as ‘peak risks’, insurance market jargon for the natural disasters that cost insurance companies hundreds of millions in payouts.

The rationale from both sides is to provide an element of protection against the very large exposures that the insurers have to such infrequent but costly

catastrophes and to diversify their risk. It leaves both insurers' core business unaffected.

Swaps in contrast to the other derivatives are a relatively recent innovation. The first cross-currency swap was unveiled only in 1981 although there were instruments with similar characteristics traded prior to this. The first interest rate swap to be publicly traded followed in 1982. The market in swaps grew very rapidly throughout the 1980s and the instrument became established as a class of derivative. The difference between a forward contract and a swap is that, with the swap, there is a multiplicity of cash flows. The two parties to a swap agree to exchange a set of predetermined cash flows rather than the single cash flow from a forward contract (this singularity also applies to futures and options). The development of an agreement that exchanged a series of cash flows helped financial market users to manage the risks of a given cash stream. As a result, market users can now swap cash flows from equities and commodities as well as manage interest rates and currencies. Additional non-standard features have been introduced to meet special circumstances, such as swaps which have option elements and are callable or puttable.

### The Risk Management Product Set

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The different derivative instruments that are traded in financial markets are often called the risk management product set because their main function is to transfer risks. The market for derivatives deals principally with market risk (or the risk that the price of the underlying variables will change over time) but other risks, such as credit risks and catastrophic risks, are also traded. The market in derivatives can be seen as a market in risk. By appropriately trading the instruments, market participants can exchange risks and reduce their exposure to undesirable economic factors. Instruments exist to manage interest rate risks, currency risks, equity risks, and commodity risks as well as some other specialised risks. The instruments used to manage these risks are:

**Forward contract:** A commercial contract between two parties to buy and sell at a price agreed today which has the delivery or settlement of the contract deferred until some mutually agreed date in the future (when the exchange then takes place). Quantity and quality are specified when the forward contract is initiated. Any contract where the delivery or settlement is later in time than that which is normal for the market in the physical commodity, known as the spot market, is a forward contract.

**Futures contract:** Functionally this is the same as the forward contract. However, it differs because the contract is traded on an exchange, the contracts are standardised for all users to facilitate trading, the contract will be between the buyer and the exchange's clearing house and the seller and the exchange's clearinghouse. The result is that the credit risk will be intermediated. In addition, both buyer and seller will be required to post a performance bond to ensure that they can fulfil their obligation under the contract.

**Option contract:** This gives the holder (or buyer) of the option the right but not the obligation to buy or sell the underlier at a specific price at or before a specific date. While the option buyer (or holder) has the right to complete the contract or not, the option seller (or writer) is obliged to complete the contract if the holder requests it.

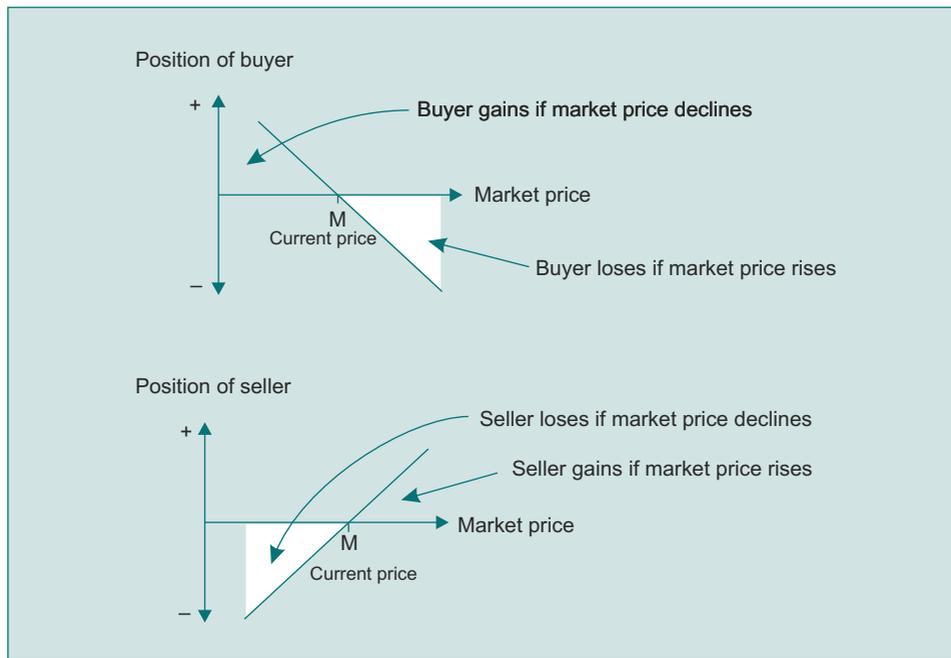
**Swap contract:** An agreement between two parties to exchange (or in financial parlance, to swap) two different sets of future periodic cash flows based on a predetermined formula.

### 1.1.1 Fundamental Financial Instruments

Fundamental financial instruments exist in order to allow individuals to invest for the future, to allow individuals and firms to raise capital and to borrow. In doing so, these instruments or securities have a number of risks. For example, investors in a firm's shares are hoping that the management will be able to realise a profit. The managers may be spectacularly successful or woefully unsuccessful in this regard. In addition, the legal and economic structure of fundamental financial instruments is designed to allow investors to modify and transfer risks as well as to address contractual problems. An investor who holds all his wealth in just one company is exposed to the risk that the business might underperform – or even fail. By creating a company where ownership is split into shares, investors can spread the risk across a great many companies. At the same time, firms can raise money from a large number of individuals. By spreading their investment across a wide range of firms investors can diversify and hence reduce the impact on their wealth of one particular business failing. As a consequence, they can take more risk in their portfolios. The legal contract also protects shareholders so that in the event of failure the most they can lose is the money they invested. These contractual arrangements help savers and borrowers to contract together and undertake economic activity.

On the other hand, derivatives are securities that obtain their existence from the value of fundamental financial instruments. They mimic the performance of the underlier. But unlike fundamental financial instruments, which are a necessary part of the economic system, derivatives are redundant securities. For a firm to raise capital, it will have to issue shares or borrow money. In theory all the benefits of derivatives can be achieved through the use of fundamental financial instruments. The reasons derivatives exist is that they provide an efficient solution to the problems of risk transfer. Take the situation where a merchant wants to lock-in the price of grain. The fundamental financial instrument solution would be to buy the grain today and store it. For most businesses this is both costly and inefficient. Far better to be able to buy in the forward market and lock in both delivery and price today in anticipation of future need. Similarly with a seller: a farmer may wish to take advantage of current high prices to lock in the selling price. Without the existence of a forward market in his produce, this is impossible. So although derivatives are technically redundant, they exist because they allow economic agents' needs to manage their risks in an efficient manner. They are the least-cost solution to the risk management process.

They also exist because there is a two-way market in risks. A buyer is exposed to potential price increases, a seller to possible price declines. We can show their positions in terms of risk profiles, as shown in Figure 1.1. The buyer and seller are both exposed to the risk that the market price will change. For the buyer the main concern is that the price will rise and future purchases will cost more. For the seller, the main concern is that the price will fall and a future sale will generate less revenue. The solution is for buyers and sellers to exchange their risks. This is what derivatives are largely designed to do. That said, as with fundamental financial instruments, derivatives can also be used – and are used – for other purposes: for investment and speculation.



**Figure 1.1 Risk profiles of buyers and sellers**

Buyers will gain if market price falls, but lose if price rises. Sellers will gain if market price rises, but will lose if price falls

In the jargon of financial markets, the buyer would be considered to be short the risk, or having a short position in the risk (or the market for the risk); the seller would be considered to be long the risk, or having a long position in the risk (or the market for the risk).

## 1.2 Arbitrage Relationships

A key issue is how to determine the value of derivative instruments. By value one means the price at which the agreement is reached (for instance the forward price for delivery) and/or any payment required by one party to the other (this applies to options). Prices of such instruments are set by arbitrage conditions. As discussed in the previous section, derivatives are functionally redundant since they can be replicated through the use of fundamental financial instruments. Consequently, the value relationships that apply between fundamental financial instruments have a

critical role in determining the value of derivatives. Of equal importance is the ability of market participants to create replicating portfolios using combinations of instruments to mimic the value of derivatives. This ability to replicate allows market participants to arbitrage between fundamental financial instruments (that is items traded in the spot markets) and derivatives.

In an economically efficient market, assets or combinations of assets that have the same payoffs should trade at the same price. In economics, classic deterministic arbitrage involves market participants buying an asset at one price in one market and simultaneously selling it at a higher price in another market thus enabling the arbitrageur to realise an immediate risk-free profit.<sup>1</sup> The rule of thumb is to buy low and sell high. For instance, if the exchange rate for sterling against the US dollar in London was \$1.75/£ and in New York it was \$1.74/£, in the absence of any market imperfections which prevented it, an arbitrageur could sell pounds in London and obtain \$1.75 and buy pounds in New York at \$1.74 netting a profit of 1¢ per pound with little or no risk.<sup>2</sup> In an efficient market such, as that which characterises foreign exchange, opportunities to arbitrage should be rare to non-existent. Economists refer to the relationship where assets, or combinations of assets, which have the same payoffs and hence should trade at the same price as the *Law of One Price*. Arbitrage ensures that prices between different assets (and combinations of assets) remain in the correct value relationship to each other.

It may take some thought and analysis to determine whether the price of two assets or combinations of assets are in the correct arbitrage-free relationship to each other. To be sure that the prices offer an arbitrage opportunity we need to know what the prices should be. Hence, we need a pricing or valuation model. In finance most models are valuation models since we want to know whether the asset, security or portfolio is being valued correctly. That is, we want to measure our should be (or theoretical) price against the actual market price.

For instance, if the current or spot market gold price is \$400 per ounce, the forward market price with one year delivery is \$450 per ounce and the one-year interest rate in US dollars is 4 per cent is there the possibility for arbitrage or are prices in the correct relationship to each other? Or what if the spot gold price is \$400/oz, the one-year forward price is \$400/oz and the one-year US dollar interest rate is 4 per cent, does this present an arbitrage opportunity? In order to answer this, we need to be able to set up a replicating portfolio to take advantage of any mispricing. The arbitrageur would need to know if any element was mispriced. In order to know whether the forward price was correct or not he would need a pricing model with which to compare the actual price. For forward contracts the theoretical price (as determined by the pricing model) is called the cost of carry. This is discussed in detail in Module 3.

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- <sup>1</sup> Note that in practice there may be some small residual risks involved. Also, the terms arbitrage and arbitrageur have been much abused. Many speculative activities, such as betting on the outcome of mergers and acquisitions, are termed arbitrage. Risk arbitrage as such activities are known has little in common with the classic definition of deterministic arbitrage.
  - <sup>2</sup> Moving money from London to New York and back is virtually costless. The counterparties to the transactions might worry about the arbitrageur's credit standing but otherwise without the presence of government regulations there is little to stop a market participant from exploiting the opportunity. Hence in a competitive market it is unlikely to be present for long.

Suffice at this point to explain that with the forward price of gold at \$450/oz, the arbitrageur would want to buy the gold in the spot market, finance this by borrowing dollars at 4 per cent and simultaneously agreeing to sell gold in one year's time. The payoff from this strategy, which is known as a cash-and-carry arbitrage, is shown in the upper half of Table 1.1. On the other hand, with the forward price of gold in one year at \$400/oz the arbitrageur would want to undertake the opposite strategy: borrow gold for a year and sell it, investing the proceeds at 4 per cent and agreeing to buy gold in the forward market. This is known as a reverse cash and carry and is shown in the lower half of Table 1.1.

**Table 1.1 Arbitrage operations in gold**

<b>Cash-and-carry in gold</b>	<b>\$</b>
<i>At initiation</i>	
Sell gold in forward contract @ \$450/oz	
Buy gold spot at \$400/oz	(400.00)
Finance purchase by borrowing for 1 year	400.00
Net investment	<u>0.00</u>
<i>At maturity</i>	
Sale of gold through forward contract	450.00
Repayment of borrowed funds	(400.00)
Interest on funds at 4%	(16.00)
Net profit	<u>34.00</u>
<b>Reverse cash-and-carry in gold</b>	
<i>At initiation</i>	
Buy gold in forward contract @ \$400/oz	
Sell gold spot at \$400/oz	400.00
Invest by lending for 1 year	(400.00)
Net position	<u>0.00</u>
<i>At maturity</i>	
Purchase gold through forward contract	(400.00)
Loan	400.00
Interest on loan	16.00
Net profit	<u>16.00</u>

Note: it is possible to configure the transaction so as to extract the profit at initiation.

Note that the cash-and-carry and reverse-cash-and-carry strategies require us to set up replicating portfolios using fundamental financial instruments. These portfolios involve buying or selling in the spot market, borrowing or lending, and taking the opposite position in the derivative.

If the price is above this replicating price we can expect many market participants to set up cash-and-carry transactions and seek to buy gold in the spot market and

sell it in the forward market. Supply and demand will push down the price at which buyers are prepared to transact in the forward market. In the same way, with the price of the forward contract at \$400/oz, market participants will seek to sell gold in the spot market and buy it back in the forward market. The only price that would prevent arbitrage is one where the forward price exactly equalled the replicating portfolio price, namely \$416/oz.

To summarise: in order to determine whether arbitrage is possible we need a pricing model for the derivative that explains what the price should be. Equally, we can consider that the only appropriate price for the forward contract is the price that prevents arbitrage. Another way to look at it is to see that the correct (or theoretical) price is the reproduction cost of taking the opposite side of the transaction. Knowing this provides a way of valuing such contracts. Another example will help to make this latter point clear. In Table 1.2, we have the exchange rate and relevant interest rates between sterling and the US dollar. At what rate would a bank agree to undertake a forward foreign exchange transaction with a customer who wished to buy £1 million and sell US dollars in 12 months' time?

**Table 1.2** Currency and interest rates for the US dollar and sterling

Market conditions	
Spot exchange rate	US\$1.4500 = £1
Interest rates	
1 year US dollar	4.00%
1 year sterling	5.00%

The reproduction approach requires us to create a replicating portfolio that is risk-free to the bank. The agreement involves the bank paying (a) £1 million and (b) receiving US dollars in exchange. We can do this by the bank (1) borrowing US dollars in the money markets for one year, (2) buying the present value of £1 million and selling dollars at the spot exchange rate and (3) depositing the sterling in the money market for one year. At maturity, the deposited sterling (3) is repaid and is used to pay (a) £1 million to the customer in exchange for which the customer gives (b) US dollars which are then used to pay off (1) the dollars borrowed by the bank. By correctly pricing the forward foreign exchange contract and trading through the replicating portfolio, the liability is exactly matched. The bank needs to quote a forward exchange rate of US\$1.43619 to the pound. The replicating transactions are shown in Table 1.3.

**Table 1.3** Replicating transactions required to price a forward foreign exchange contract

At initiation				
	US dollars	Exchange rate	Sterling	
Borrow \$	1 380 952.38	\$1.4500	(952 380.95)	Invest £
	[1]	[2]	[3]	
At maturity				
Customer pays	(1 436 190.48)	\$1.4362	1 000 000	Customer receives
	(b)		(a)	

Notes:

(a) customer buys sterling and (b) sells US dollars

[1] bank borrows US dollars at 4 per cent per annum which will be offset at the maturity of the forward contract by the customer delivering US dollars (b)

[2] bank converts US dollars into sterling at the spot exchange rate of \$1.4500

[3] bank invests the sterling at the one-year sterling rate of 5 per cent. At maturity, sterling will be used to pay the customer (a)

In practice, the bank can simply price the forward foreign exchange contract using the interest rate parity relationship for the forward foreign exchange rate:

$$f_t = S \frac{(1+r_f)^t}{(1+r_d)^t} \quad (1.1)$$

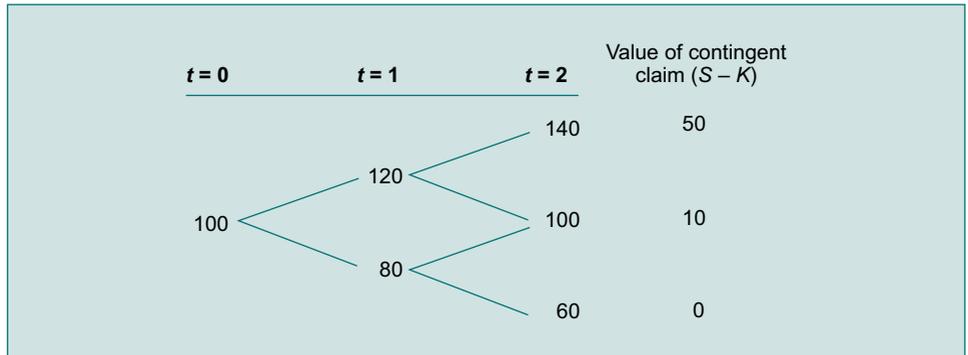
where  $f_t$  is the forward rate at time  $t$ ,  $r_f$  and  $r_d$  are foreign (quoted currency) and domestic (base currency) interest rates respectively for the currency pair for the time period  $t$ . Equation 1.1 gives the same result as the replicating portfolio calculations in Table 1.3 and can be considered an arbitrage-free pricing model for the forward foreign exchange rate. In fact, the interest rate parity model is a variant of the cost of carry model discussed earlier in the context of the gold price, which is also, as we have seen, an arbitrage-free pricing model.

### 1.2.1 Dynamic Arbitrage

Not all arbitrage operations can be undertaken simultaneously. Consider the following situation. Take a contingent claim (an option to purchase a share) which has an agreed purchase price of \$90 after two years. The current share price is \$100. We don't know what the price of the shares will be in two years' time. We do know that if the share price is less than \$90, the holder of the contingent claim will not exercise their right of purchase and instead will buy at the then prevailing lower market price. Given this uncertainty, we cannot simply buy the shares now and sell them to the contingent claim holder at maturity. Let us now assume that an investor is willing to pay \$25 for this contingent claim. Is there an arbitrage opportunity?

We need to know something about how the share price might behave between now and two years' time. Keeping things simple, we know that at  $t=1$  the share

price might rise to \$120 or it might fall to \$80. If it rises to \$120 at  $t = 1$ , at  $t = 2$  it might subsequently rise again to \$140 or fall back to \$100. If on the other hand it falls to \$80 at  $t = 1$ , at  $t = 2$  the price might recover to \$100 or continue its fall to \$60. The possible price paths for the share are given in Figure 1.2.



**Figure 1.2** Possible price paths for the share

The value of our contingent claim will therefore depend on the possible price paths between now and year 2. Its current value is the difference between the market price for the shares and the price at which the claim can be exercised. The current price is \$100 for the shares and the price at which the shares can be purchased is \$90, so the claim must be worth at least \$10. That is the claim must be worth a minimum of  $(S - K)$  where  $S$  is the share price and  $K$  is the price at which the share can be purchased. Since the contingent claim is an option, if the share price is below  $K$ , the investor will not exercise the right of purchase and abandon the claim. So the payoff  $(S - K)$  is bounded on the downside at zero. The payoff will be the maximum of  $(S - K)$ , or zero. Depending on the future price behaviour the value of the contingent claim will be:

Share price at $t=2$	140	100	60
Contingent claim value	50	10	0

As with the earlier examples, the arbitrageur will want to sell the overpriced element and hold the correctly priced one. In this case it involves selling the contingent claim and holding the arbitrage or replicating portfolio. At initiation, the arbitrageur will have sold one contingent claim and will take a fractional investment of 0.6985 shares plus borrowing 48.32.<sup>3</sup> Interest rates are 4 per cent per annum. The position at  $t=0$  is given in Table 1.4.

<sup>3</sup> The fractional investment, known as delta ( $\Delta$ ), is determined by the ratio of price change in the derivative if the share price rises or falls to that of the underlier, namely:

$$\Delta = \frac{C_U - C_D}{S_U - S_D} \tag{1.1}$$

The share price range is  $120 - 80$  and the value of  $\Delta$  is .6985 so the value  $(C_U - C_D)$  is 27.94. To solve for  $\Delta$  we need to know the value of the contingent claim at  $t=1$  for both the up move (U) and

**Table 1.4 Arbitrage position at t=0**

Component	Value
Buy 0.6985 of a share	(69.85)
Borrow	48.32
Sell contingent claim	25.00
Net position/gain	3.47

Arbitrageur has sold one contingent claim and set up a replicating portfolio to deliver the commitment to sell, if required, under the claim

What happens at the end of year one? The arbitrageur does not know whether the share price will go up or down. However, the portfolio will need to be rebalanced at t=1. After one year, if the share price has risen, the required fractional holding needs to be increased (in this case to one, or one share). If the share price has fallen, then the fractional holding needs to be reduced (in this case to 0.25 or a quarter of a share). The net value of the position at t=1 when the share price has either risen or fallen and after rebalancing is shown in Table 1.5.

**Table 1.5 Arbitrage position at t=1**

Component	Share price	
	120	80
Value of fractional holding in share from t=0	83.82	55.88
Required fractional holding in shares	1.000	0.2500
Required additional holding	.3015	(.4485)
Adjustment to share position	36.18	(35.88)
[A] Total position in shares	120.00	20.00
Original borrowing (48.32) plus interest at 4%	50.29	50.29
Additional borrowing/(repayment)	36.18	(35.88)
[B] Net borrowed funds	86.47	14.41
[A – B] Value of position (contingent claim)	33.53	5.58

Arbitrageur rebalances the replication portfolio established at t=1. If the value of the shares has risen, the arbitrageur increases the fractional holding in the shares; if the share price has fallen, the arbitrageur reduces the fractional holding in the shares.

the down move (D). We can only find this by solving first the value of the claim at t=2 and working backwards to find the theoretical (or arbitrage free) price of the claim at t=1, knowing its value at t=2. The value of the position in Table 1.5 in the upper node is 33.53 and the lower node is 5.58. So for t=0, the appropriate fractional investment to take in the share is:

$$\Delta = \frac{33.53 - 5.58}{120 - 80} = .6985$$

At maturity, the contingent claim's value will depend on how the share price has performed between the first and second year. As Figure 1.2 shows there are three possible outcomes. The result of the replicating portfolio is given in Table 1.6.

As Table 1.6 shows by following the replicating strategy, regardless of the outcome at maturity, the arbitrageur has exactly the required amount of money to pay off the value of the contingent claim. With the share price at \$140 and the exercise price of \$90 the contingent claim seller has to deliver a security worth \$140 for \$90. Buying the security in the market at \$140 but selling at \$90 means a loss of \$50. The replicating strategy has delivered a profit of \$50 so the arbitrageur walks away without loss.

**Table 1.6** Arbitrage position at  $t=2$

Component	Share price		
	140	100 <sup>1</sup>	60
[A] Portfolio from $t=1$ when share = \$120	140	100	
[B] Borrowing (86.47) plus interest at 4%	90	90	
[A – B] Net value of position	50	10 <sup>(a)</sup>	
[A] Portfolio from $t=1$ when share = \$80		25	25
[B] Borrowing (14.41) plus interest at 4%		15	15
[A – B] Net value of position		10 <sup>(b)</sup>	0
Payout on contingent claim	50	10	0
Net position of arbitrageur	0	0	0

Possible outcomes depending on the share price at  $t=1$

1 Note that either outcome (a) or (b) occurs depending on what happens at  $t=1$

As with the earlier examples for gold and the forward foreign exchange transaction, the theoretical or arbitrage free price thrown up by the model for the contingent claim is the price that exactly compensates the contingent claim seller for replicating the payoff of the claim. This means the correct theoretical price for the contingent claim should have been \$21.53. Market forces will lead arbitrageurs to sell contingent claims if the market price is above the theoretical price and buy them if it is below thus forcing convergence to the theoretical price.<sup>4</sup>

The model for valuing a contingent claim is known as a conditional arbitrage model and requires the arbitrageur to rebalance the replicating portfolio as the value of the underlier changes. This conditional arbitrage model is the basis of all standard

<sup>4</sup> As with the cost-of-carry example, if the price of the contingent claim is below that of the replicating portfolio, the arbitrageur will buy the contingent claim and sell the replicating portfolio (going short the shares and lending) and rebalancing at  $t=1$ .

option pricing models where the value of the option is determined by reference to its replicating portfolio. For this reason such pricing models are often called arbitrage pricing models.

An important corollary of the replicating portfolio approach is that the contingent claim seller, who has the obligation to deliver under the contract, is indifferent to the price behaviour of the underlier. Hence risk preferences do not affect the pricing of these claims. As the position is risk-free, it will earn the risk-free rate of interest and this means that complications about risk-adjusted discount rates can be ignored when working out the present value of the portfolio.

Note another outcome of the modelling process: even without an arbitrage opportunity, the dynamic replication strategy allows the contingent claim seller to manage the risk from selling the contract. In the example above, once the vendor has received \$21.53 for the contingent claim, by following the dynamic replication strategy, the writer has eliminated all risk.<sup>5</sup>

## 1.2.2 Impediments to the *Law of One Price*

In an efficient market there are no impediments to prevent smart market participants exploiting the fact that if there are two assets or packages of assets that have the same payoff and which have different prices then arbitrage can be undertaken. In order to determine whether there is a profitable arbitrage opportunity, the market participant may have to undertake sophisticated modelling to determine whether he can construct a replicating portfolio synthetically via a combination of fundamental financial instruments. Given the potential rewards from arbitrage, market participants will devote time and effort to constructing replicating portfolios in order to exploit incorrect prices.

How realistic is it for market participants to undertake such arbitrages? The replicating portfolio is almost the same as the asset or contingent claim being replicated. To the extent that the model has non-realistic assumptions when applied in practice then the values of the two may differ. Arbitrageurs and contingent claim vendors are always seeking to improve the accuracy of their models. However, the real world departs from that of the models. In particular, transaction costs affect the result and are not a feature of most theoretical models. In the case of our dynamic replication example, the arbitrageur does not know in advance whether at  $t=1$  more shares will be purchased or sold – and how many. Hence transaction costs will affect the exactness of the result.

Other real world market imperfections or frictions can also affect the result. One possible problem is contractual uncertainties. For instance, when a market participant sells shares he does not own, these have to be borrowed. Generally shares can only be borrowed for a short period (days or weeks). Hence the maturity of the

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<sup>5</sup> In practice of course the model is only a representation of reality and to the extent that actual market behaviour differs from that assumed in the model the writer will have an element of residual risk. Hence a prudent writer will charge more for the option to cover himself. But to the extent that actual and model behaviour converge, competition for business in financial markets will drive down the prices of contingent claims towards their theoretical values.

contingent claim being replicated may differ from the transactions that underpin the replicating portfolio. There are other complications from stock borrowing. The stock lender may require a haircut (or prudential deposit) so that the short seller does not receive the totality of the value of the short sale. Also, it is the case that borrowing and lending rates differ.

Another issue is taxes. The assumption of most models is that there are no taxes. In practice, the tax treatment of the gains and losses from the written leg of the position (the contingent claim sold to the investor) might be treated differently from that of the components of the replicating portfolio. So one may not be able to offset the other leading to unanticipated losses. Another factor is the periodic apparent irrationality of financial markets. For instance, in periods of disturbance or stress, pricing relationships can break down leading to unanticipated losses.<sup>6</sup> Yet another factor that can make arbitrage hazardous are differences in information between market participants. Prices at which transactions are made may not reflect the true intrinsic value of the instruments being traded.

The result is that while the pricing models that are used to compute the theoretical or fair value of a derivative have been shown to be good representations of the actual market prices of such instruments, the models are not quite the same thing as the derivatives themselves. This always needs to be kept in mind when considering the analysis of such models. Nevertheless, the arbitrage principle is a powerful tool for both analysing derivatives and explaining the observed prices of such instruments in financial markets.

### 1.3 Derivative Markets

Derivatives are traded in financial markets. We can distinguish two types of markets and instruments: exchange-traded and over-the-counter (OTC) markets. Exchange-traded instruments are bought and sold through an organised exchange. For example, in the UK, the major exchange for financial derivatives is *Euronext-LIFFE*. On this exchange, interest rate, equity and commodity derivatives are traded. In order to facilitate trading products are standardised. For instance, all the options traded on a particular underlier will have the same terms. The exchange will fix the number of units in the underlier, the maturity dates and the exercise prices for the options, where and when the underlier is to be delivered. These are laid out in the contract specifications. The only factor that will vary will be the price at which the options trade. Transactions either are executed on the trading floor or – as with *Euronext-LIFFE* – through screen-based trading systems. The exchange controls how trading is organised and regulates the activities of traders, who have to be registered with the exchange. The way trading and other elements of the settlement process are organised means that market participants have virtually no credit risk.

In contrast OTC markets involve bilateral transactions between market participants. Since these are negotiated directly between the parties involved, it is possible

<sup>6</sup> A good example is the collapse of Long-Term Capital Management (LTCM). See Roger Lowenstein (2001), *When Genius Failed*, New York: HarperCollins Publishers.

to offer non-standard products. Unlike exchange-traded products that have to be standardised to ensure liquidity, OTC markets can offer great flexibility to users. All terms can be negotiated and customised to meet the needs of the parties involved. However, since contracts are negotiated directly between the two parties, OTC transactions are subject to credit risk. This means that only counterparties with a good credit reputation are acceptable as counterparties although mechanisms similar to those used for exchange-traded products can be used to alleviate this problem. Plus since the contracts are customised, it is not easy to unwind or cancel such a contract after it has been agreed.

### Derivatives Markets Terminology

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Derivatives markets are replete with their own, sometimes esoteric, terminology. Some of the more common terms are given below.

**Cash market:** The market in fundamental financial instruments or physical goods. Also called the spot market.

**Derivative or derivative instrument or security:** A contract whose payoff and hence value is determined by the price of another underlying asset. Also referred to as a contingent claim.

**Contract specifications or characteristics:** The terms detailing the quality, size, price and delivery terms of a derivatives contract. For over-the-counter markets these might differ between transactions, for exchange-traded contracts only some elements are negotiable.

**Underlier:** the fundamental financial instrument, portfolio, or physical asset, from which the derivatives contract obtains its value.

**Delivery:** Procedures for settling the payment and receipt of the underlier at maturity or upon exercise. Some contracts do not involve a transfer of the underlier from seller to buyer and settle by paying the difference between the contracted price and the delivery price.

**Clearinghouse:** The institution which, as its name suggests, organises the settlement of transactions and, for exchange-traded derivatives, acts as the counterparty to all transactions.

**Long or long position:** A situation where a market participant either currently holds the underlier or will need to purchase the underlier in the future. Hence a purchaser of a forward or futures contract who is contracted to receive the underlier at the maturity of the contract is deemed to have a long position in the contract.

**Short or short position:** A situation where a market participant either currently has sold the underlier short or will need to sell the underlier in the future. Hence a seller of a forward contract who is contracted to deliver the underlier at the maturity of the contract is deemed to have a short position in the contract.

**Holder.** The buyer of an option. The buyer has the right to exercise the option and complete the transaction if it is advantageous to do so. That is, the buyer 'holds' the rights from the option.

**Writer:** The seller of an option. With a call option the writer has to sell at the strike price, with a put the writer has to buy at the strike price. Hence the seller has written the right of exercise.

**Exercise:** To activate the right to purchase or sell given by an option.

**Exercise price or strike price:** The contracted price (or rate) at which an option holder can execute or complete the transaction.

**Expiration:** The point at which a derivatives contract ceases to exist, that is it expires. Also called maturity.

**Life:** The length of time a derivatives contract is in force. Also called the tenor.

## 1.4 Uses of Derivatives

As discussed above in Section 1.2.2, market participants need fundamental financial instruments to borrow and lend. As we have seen, derivatives can be replicated using combinations of fundamental financial instruments. That said, derivative instruments provide an efficient or least-cost means of undertaking many financial activities. Their ability to meet the many different needs of market users reinforces their importance in the financial system. This section examines the different uses to which market participants put derivatives.<sup>7</sup>

### 1.4.1 Risk Modification

The fundamental justification for the existence of derivatives is their ability to modify risks. Consider the following situation. An investor can either buy a share with a current value of \$100 or purchase an option to buy the share for \$4. With the option the investor has the right to buy the share in six months' time at \$100. What are the possible outcomes? Let us assume that the share can be worth either \$120 or \$80 in six months' time. With the immediate share purchase, the investor can either gain or lose \$20 depending on the outcome. With the option, however, the investor's maximum loss is \$4, the cost of the option. Only if the share price is at \$120 in six months' time will the investor exercise his right to buy at \$100. Then his gain will be  $\$120 - \$100 - \$4$ , or \$16. By buying the option rather than the share the investor has modified his risk. The maximum loss is now limited. In like fashion all derivatives allow users to modify their risks. This ability to modify risk is a key characteristic of derivatives and justifies their position within the financial pantheon.

<sup>7</sup> Of course the total market activity recorded for particular types of derivative will represent the sum of the different uses that market participants have for the particular instruments.

Note that risk modification can involve taking more risk rather than reducing risk.<sup>8</sup> If the investor had wanted to take more risk then he could have written the option!

## 1.4.2 Hedging

Hedging is a special case of risk modification that has as objective the elimination of all risk. While risk modification changes the nature of a risk but may not eliminate it completely, with hedging the intention is to remove the source of risk. For instance, a company is selling its product abroad. The currency in which the buyer negotiates is not the operating currency of the seller. Once the contract is struck, the seller is faced with the fact that due to the time lag between agreeing terms and receiving payment there is a risk that the exchange rate will have changed. Derivatives provide a simple solution to this problem. In this case the seller can agree a forward foreign exchange contract with a bank to sell the foreign currency and buy the domestic currency. In this way the company has hedged its exchange rate risk on the sale. The intention when entering the forward contract is to reduce the unwanted exchange rate risk to as little as possible. This will be zero in this case as the forward foreign exchange contract exactly matches and offsets the foreign currency position. In other cases, the fit might not be so exact and the hedge will be imperfect. Nevertheless, the intention when using derivatives for hedging is to obtain the maximum protection from the source of risk even if there is some residual risk. With an imperfect hedge, some protection is better than none at all.

## 1.4.3 Speculation

Speculation is risk modification designed to benefit from exposure to a particular risk. Take the situation where a market participant has a view that as a result of tensions in the Gulf region, the oil price will increase. Strategy one is to buy crude oil in the spot market. There are significant disadvantages to this strategy if the only reason for buying oil is to profit from an anticipated increase in price. Oil is a bulky commodity and will have to be stored, and this can be costly. It is also necessary to find a buyer for the oil when the anticipated price increase has taken place. Far simpler from the speculator's perspective is to buy crude oil futures. That is, exchange-traded contracts that fix the price at which crude oil can be bought and sold at a specific date in the future. These have the same economic exposure to changes in the spot price for crude oil but none of the disadvantages of physical ownership. In fact by using futures, which are highly liquid instruments, the speculator can immediately take a position in the crude oil market (the underlying risk factor) and sell it again without worrying about finding a seller, storage or an eventual buyer. The costs of setting up a position to take advantage of a rise in the crude oil price in the futures market will be far less than the costs of setting up a similar trade in the spot market. That means that even a relatively small increase in the crude oil price will make money for the speculator. So while derivative markets

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<sup>8</sup> The emphasis of this course will be on how derivatives can modify risk and, in particular, how they are used for risk management. This focus builds on the principles and processes of the eMBA elective *Financial Risk Management*.

are designed to manage risks, they do allow the more adventurous to benefit from assuming risk. In this case, the speculator is taking the risk that oil prices do not rise as anticipated! The existence of speculative activity in derivative markets acts to increase the pool of capital available for the market and to increase the supply of counterparties thus increasing the market size for market participants who are natural hedgers.

#### 1.4.4 Arbitrage

Arbitrage operations aim to exploit price anomalies. The basic mechanisms have been described in Section 1.2. The existence of derivatives provides arbitrageurs with more pricing relationships that can be exploited if the prices move away from their correct relationships. For instance, if the prices at which options are traded differ from their theoretical value, arbitrageurs will step in to exploit this fact. Take the situation where a call option on a share with an exercise price of \$100 is trading at \$4.5 and the corresponding put (with the same exercise price) is trading at \$2.7. The current share price is \$102. The options have 3 months to maturity (expiration) after which they are void. The three months interest rate is 4 per cent per annum. A trader can arbitrage the mispricing of the call and the put. The trader buys the call for \$4.5, sells the put at \$2.7 and sells the share for \$102 and invests the present value of the \$100 exercise price (this is \$99.02). The net gain from this is \$1.18. At maturity one of two situations arises. If the share price is above \$100, the arbitrageur exercises the call by using the invested funds and receives the share. This share is then returned to the stock lender. If the share price is below \$100, the call is abandoned. Having written the put the arbitrageur is now contractually committed to purchasing the share for \$100 when its market price is less than this. The holder exercises the put and the arbitrageur pays for the share he is obliged to receive using the invested funds. Again the share is returned to the stock lender. Whatever the outcome, the arbitrageur nets a \$1.18 from the transaction without having to invest any of his own money.

#### 1.4.5 Spreading

Spreading involves taking advantage of – or limiting the impact of – price changes between two assets. Hence it can be either for speculative purposes or for risk management. Extending the oil speculator example in Section 1.4.3, now the speculator has a view that the margin between unrefined crude oil and its refined products (unleaded gasoline and heating oil) is likely to increase due to refining capacity shortages. He wants to take advantage of this fact. One possibility, as with the simple directional crude oil transaction, is for him to sell crude oil and buy unleaded gasoline and heating oil in the spot market. But this is even more complicated than the simple strategy of buying crude oil on the expectation that its price will rise. It is far simpler for the speculator to deal in energy futures contracts. Contracts exist for crude oil and its refined elements, unleaded gasoline and heating oil. By buying futures in the refined products and selling the crude oil futures, the

speculator is anticipating a widening their price relationship.<sup>9</sup> If he is correct, regardless of whether the crude oil price goes up or down, the speculator will make money. This is an important feature of spread trading. The profit (or loss) from the spread transaction is not dependent on absolute price levels but on changes in the price relationship between the two assets. Using derivatives for spreading reduces the cost and complexity of setting up transactions designed to exploit or hedge changes in this relationship. Hence derivatives are the instruments of choice for this type of transaction. In fact, to use the spot or physical markets to exploit these spread relationships, speculators would have to anticipate very significant changes in their relative prices to compensate for transaction and other costs.

### **1.4.6 Decreasing Financing Costs**

Derivatives allow users to modify their risks. They can help firms decrease their financing costs. Take the situation where a company can either borrow in its own country and lend the money to its foreign subsidiary or the subsidiary can borrow in the local currency. In the case where the subsidiary borrows locally, the local income will service the debt. In the case where the parent supplies the funds, the local income has to be exchanged for the currency of the parent company. The company is likely to be able to borrow in its own country on much finer terms because it is better known and respected than in the foreign country where it is less well known. By borrowing locally it is paying more (but eliminating the exchange rate risk on the borrowing). The company would benefit if it could borrow in its home country and yet lend in the local currency of its subsidiary. This is precisely what cross-currency swaps allow firms to do. They can raise finance in the cheapest market and currency without having to worry about the exchange rate risk. The cross-currency swap converts the borrowed currency into the desired currency while at the same time eliminating the exchange rate risk. Firms can reduce their financing costs because derivatives are available to manage undesirable financial risks.

### **1.4.7 Tax and Regulatory Arbitrage**

Under UK laws, individuals and firms have the right to organise their affairs to minimise the amount of taxes they pay. Derivatives allow firms to manage their tax liabilities. For instance, a firm that borrows money from a bank may not know what its future interest rate cost will be. Interest expense is normally tax deductible, but only if there is sufficient profit. The firm therefore may be exposed to unanticipated increases in borrowing costs that it cannot offset against its profits if these are not large enough. It may therefore want to fix the total interest charge it pays so as to ensure that it can take advantage of the interest rate tax shield. The firm can do this using derivatives. By entering into a forward rate agreement the firm can fix the amount of interest it will pay for a given period without having to renegotiate its

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<sup>9</sup> This margin is known as the crack spread. That is, the difference in price between the unrefined and refined products which represent the refiner's costs and margin from 'cracking' the crude into its constituents. A refiner might be interested in protecting this margin and hence would undertake a crack spread designed to lock-in a fixed margin if it was of the view that excess refining capacity was likely to depress margins.

borrowing from the bank. The contract converts an uncertain future interest expense into a fixed or certain expense.

In a similar manner to the firm's management of its tax deductibles, banks and other regulated financial institutions can manage the amount of regulatory capital required to support their business. One way of doing this is to use derivatives. Banks have to allocate more capital against loans to commercial enterprises than for loans to governments and state entities to cover against the potential default risk. This means banks are limited in the amount of lending they can make to commercial firms without raising more capital. Banks can use derivatives to reduce the amount of capital required to lend to commercial enterprises. By using a credit derivative, the bank buys insurance against default. As a result financial regulators are prepared to allow banks which have lent to commercial enterprises and used credit derivatives to transfer the default risk to allocate less capital to such loans.

### 1.4.8 Completing the Market

Finance theory suggests that it should be possible to construct unique payoffs for every future possible state of the world. Take the simple example given in Table 1.7 that assumes there are only two possible outcomes and two available securities.

Security	Price	State of nature	
		1	2
B	9	10	10
M	13	10	20

By holding judicious combinations of security B and M, a financial engineer can create portfolios which have a positive payoff in one state and zero payoff in the other. The cost of setting up a security with a payoff of 1 in state two will be 0.4 and that for state one will be 0.5.<sup>10</sup>

In that sense, the two available securities B and M 'span the market' and the market can be considered complete. If, on the other hand, as given in Table 1.8,

<sup>10</sup> The replicating portfolio will be created by finding the appropriate value for  $\Delta$  such that the portfolio is risk-free with in state one a net value of zero and in state two a net value of one. This is obtained by:

$$\Delta = \frac{1 - 0}{20 - 10} = .1$$

And setting up the replicating portfolio, such that:

$$\begin{aligned} 20\Delta - (1 + r_f)B &= 1 \\ 10\Delta - (1 + r_f)B &= 0 \end{aligned}$$

Where B is the amount of borrowing at the risk-free rate (which is 11.11 per cent). The cost now of setting up such a portfolio will be:

$$13\Delta - B/(1 + r_f) = .4$$

In like fashion, the cost of setting up a replicating portfolio with a payoff of 1 in state one and zero in state two is 0.5.

there are three states of nature and only two securities, then the market is incomplete.

**Table I.8** Incomplete market

Security	Price	State of nature		
		1	2	3
B	9	10	10	10
M	13	9	11	13

The market could be completed by adding a third security, such as a derivative that had a positive payoff in one of the states but a value of zero in the others. Then the available securities would span the market and it would be complete and it would be possible to construct a replicating portfolio that generated a positive value in one state and zero in the others.

While this analysis is largely theoretical and based on a simple example, the ability of derivatives to help complete the market provides an important justification for their existence.

## I.5 Learning Summary

Derivatives are contracts specifically designed to manage risks. Although technically redundant securities since they can be replicated using fundamental financial instruments, they provide an efficient means for market participants to manage and transfer risks. Their importance in this role continues to increase and they have become an important element in modern financial markets. While some, such as futures and swaps, are relatively new classes of instruments others such as forward contracts and options have always been a feature of commercial life. The great expansion over the last 30 years or so in derivatives on fundamental financial instruments is due to changes in the financial system and theoretical developments in our understanding of how these instruments can be valued.

A key principle of valuation in an efficient market is the ability of replicating portfolios made up of fundamental financial instruments to provide the same payoffs as derivatives. Under the *Law of One Price*, two assets or combinations of assets with the same payoffs should have the same price. This identity between the derivative contract and a replicating portfolio with the same payoffs as the derivative is enforced by arbitrage. While this theoretical understanding provides the ability to price derivatives, frictions in real world financial markets may lead to divergences between theoretical arbitrage-free prices and actual market prices for derivatives.

Derivatives are traded either on organised exchanges with specific rules and a significant degree of investor protection or directly between market participants in the over-the-counter markets. In the later case, market participants have to take into consideration the credit risk of the counterparty to the transaction. Exchange-traded contracts have standardised terms and conditions, OTC derivatives can be customised as required.

Derivatives provide market participants with not just the opportunity to modify risks, but also to engage in speculation and to undertake transactions that would otherwise be problematical when undertaken using fundamental financial instruments. These include such benefits as reducing financing costs and taking advantage of tax benefits and regulations.

## Review Questions

### Multiple Choice Questions

- 1.1 Which of the following is correct? The forward market that existed in the Netherlands at Antwerp in the 14th century was a market for:
- A. Grain and other agricultural produce.
  - B. Tulip bulbs.
  - C. Currencies.
  - D. Wool and cloth.
- 1.2 Which of the following best describes the nature of a forward contract? With a forward contract, the two parties agree to:
- A. exchange an item of a specific quality for cash at a future predetermined date.
  - B. exchange an item for an agreed amount of cash at a future predetermined date.
  - C. exchange a given amount of an item for an agreed amount of cash at a future predetermined date.
  - D. exchange a given amount of an item of a specific quality for an agreed amount of cash at a future predetermined date.
- 1.3 If you have a \_\_\_\_ sensitivity to changes in market prices, you would be said to be \_\_\_\_ and would benefit from an \_\_\_\_ in the market price. Which is correct?
- A. positive    long the risk    increase
  - B. positive    short the risk    decrease
  - C. negative    long the risk    decrease
  - D. negative    short the risk    increase
- 1.4 Which of the following correctly describes a futures contract?
- A. A futures is an instrument whose value depends on the values of other more basic underlying variables.
  - B. An exchange-traded contract to buy or sell a specific amount of an asset or security for a specific price or rate on a specific future date.
  - C. An agreement to buy or sell an asset at a certain time in the future for a certain price (the delivery price).
  - D. All of A, B, and C.

- 1.5 What is the economic difference between forward contracts and futures?
- A. There is no economic difference between forward contracts and futures.
  - B. Futures are only available on some underlying assets, whereas it is possible to trade any asset with forward contracts.
  - C. Futures contracts are traded on an exchange and have standardised terms and conditions whereas forward contracts are traded over-the-counter and have negotiated terms.
  - D. Both B and C explain the economic difference between forward and futures contracts.
- 1.6 Which of the following is correct? A swap is:
- A. An agreement between two counterparties to exchange two different sets of future periodic cash flows.
  - B. The spot purchase or sale of a commodity combined with the simultaneous sale or purchase of the same commodity in the forward market.
  - C. The sale of one security to purchase another.
  - D. None of A, B, or C, correctly defines a swap.
- 1.7 Which of the following is correct? An exotic option is:
- A. an option to exchange currencies where one of the currency pair is an emerging market country.
  - B. an option which has non-standard terms and conditions.
  - C. an option-like feature that has been incorporated into a security.
  - D. an option that is not traded on a derivatives exchange.
- 1.8 Which of the following is correct? The major impediment to market participants using forward contracts is:
- A. The reputation and credit standing of the counterparty on the other side.
  - B. The lack of counterparties willing to enter the other side of the transaction.
  - C. There are no transactions available with the right maturity.
  - D. All of A, B and C.
- 1.9 Which of the following is correct? Fundamental financial instruments are:
- A. a set of redundant securities issued by firms to investors.
  - B. required by firms in order to raise capital and borrow money.
  - C. those replicating transactions used to model the payoff of contingent claims.
  - D. another name for the risk management product set.
- 1.10 Which of the following is not deterministic arbitrage?
- A. You borrow in euros and lend in dollars and buy dollars in the forward market to exploit a mispricing opportunity in the market.
  - B. In a takeover situation, you buy the target company's shares and sell the bidder's company shares to exploit a mispricing opportunity in the market.
  - C. You sell gold in London and simultaneously buy gold in Los Angeles to exploit a mispricing opportunity in the market.
  - D. You sell crude oil futures and buy crude oil in the spot market to exploit a mispricing opportunity in the market.

- 1.11 Which of the following is correct? Dynamic arbitrage requires that:
- A. the derivative that is sold and the offsetting arbitrage transactions have the same value at maturity.
  - B. the payoffs at maturity of the element that has been sold is less than that of the purchased element.
  - C. the composition of the replicating portfolio be adjusted over time in response to changes in the derivative price.
  - D. the replicating portfolio is rebalanced over time to maintain the correct relationship to the derivative being arbitrated.
- 1.12 Which of the following is the correct definition of a replicating portfolio?
- A. A package of securities and borrowing or lending designed to give the same payoff as another financial security.
  - B. A portfolio of securities designed to meet a specific investment objective or target.
  - C. A package of fundamental financial instruments and derivative securities designed to meet a specific investment objective or target.
  - D. A portfolio of fundamental financial instruments and derivative securities designed to eliminate risk.
- 1.13 Which of the following is *not* a fundamental financial instrument?
- A. A share.
  - B. A call option on a share.
  - C. A bond.
  - D. A bank loan.
- 1.14 The spot price of a commodity is \$1200 and its forward price in one year is \$1255. The one-year interest rate is 4 per cent per annum. Which of the following is correct? An arbitrageur can create a replicating portfolio by:
- A. borrowing and buying the commodity in the cash market and buying the forward contract to give a profit of \$55.
  - B. selling the commodity in the cash market and investing and buying the forward contract to give a profit of \$7.
  - C. borrowing and buying the commodity in the cash market and selling the forward contract to give a profit of \$7.
  - D. selling the commodity in the cash market and investing and selling the forward contract to give a profit of \$55.

- 1.15 The spot exchange rate between sterling and the US dollar is \$1.7425/£. The six-month interest rate in sterling is 3.75 per cent per annum and that in US dollars is 2.5 per cent per annum. The six-month forward foreign exchange rate is \$1.7385. Which of the following is correct? An arbitrageur can create a replicating portfolio by:
- A. borrowing US\$1.7425 million for six months, exchanging it at the spot exchange rate into sterling, investing the sterling, and selling sterling at the forward exchange rate to make a net profit of \$6650.
  - B. borrowing £1 million for six months, exchanging it at the spot exchange rate into US dollars, investing the dollars, and selling the dollars at the forward exchange rate to make a net profit of \$6650.
  - C. borrowing US\$1.7425 million for six months, exchanging it at the spot rate into dollars, investing the dollars, and buying the dollars at the forward exchange rate to make a net profit of \$6650.
  - D. borrowing £1 million for six months, exchanging it at the spot exchange rate into sterling, investing the sterling, and buying the sterling at the forward exchange rate to make a net profit of \$6650.
- 1.16 Which of the following is correct? In the context of derivatives markets, hedging can be considered to be a special case of:
- A. arbitrage that involves taking no risk on delivery.
  - B. risk reduction where the intention is to eliminate all risks.
  - C. speculation where the intention is to take on as much risk as possible.
  - D. financial engineering that involves taking no model risk.
- 1.17 The general rule for undertaking arbitrage is this: \_\_\_\_ and \_\_\_\_ which means, in terms of derivatives, \_\_\_\_ a derivative instrument when its price is \_\_\_\_ its theoretical or fair value price. Which of the following is correct?
- A. buy low    sell high    selling    above
  - B. sell low    buy high    buying    below
  - C. buy low    sell high    buying    below
  - D. sell low    buy high    selling    above
- 1.18 Why might you not wish to undertake an arbitrage transaction despite the fact there appeared to be a profitable opportunity available?
- A. There are uncertainties surrounding the model used to evaluate the arbitrage opportunity which might lead to a loss rather than a gain.
  - B. There are timing differences in the nature of the two sides of the arbitrage opportunity which might lead to a loss rather than a gain.
  - C. The tax treatment of the gains and losses may differ and one may fail to offset the other which might lead to a loss rather than a gain.
  - D. All of A, B and C might lead to a decision not to arbitrage an apparently profitable opportunity.

## Case Study I.1: Terms and Conditions of a Futures Contract

- I You have been asked to research and propose a new futures contract on pepper. Pepper is a consumption commodity and is a major additive to food both during preparation and at the table and, by weight, is an expensive commodity. Lay out the specifications of the contract giving all the important elements you would need to include in the contract so that potential users would know exactly what is being traded.

## Case Study I.2: Constructing a Derivative Security using Fundamental Financial Instruments

There is an economy which has only two possible futures states or conditions. Either economic conditions will be good, or they will be poor. Two fundamental financial instruments or securities exist in this economy which are used to finance operations; we can consider these to be debt and equity.

The current or market prices of the two securities or fundamental financial instruments available in the market are given below and the values that they may have in one year's time, depending on the state of the economy:

Time Security	t=0	t=1	
		Under good conditions	Under poor conditions
Security 1 (debt)	100	105	105
Security 2 (equity)	60	120	30

- I Create two derivative securities from these fundamental financial instruments.
  1. A derivative security that will provide a positive return under good market conditions, but no losses if the market at  $t=1$  turns out to be poor.
  2. A derivative security that will provide a positive return under poor market conditions, but no losses if the market at  $t=1$  turns out to be good.

*Hint:* you must think of a suitable combination or portfolio of the two securities which provides a payoff in the two states, such that in the desired state it has a positive value and zero value in the undesired state.

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